EJERCICIO (08:36)

Diagonalizar el operador Hamiltoniano de Dirac:

$$\boldsymbol{\hat{H} = \int d^3x\, \hat{\psi}^\dagger(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi} = \int \frac{d^3p}{(2\pi)^3}\sum_{r=1}^{2} E_p\left(a_r{}^\dagger{}_{(p)}a_{r(p)} - d_{r(p)}d_r{}^\dagger{}_{(p)}\right)}$$

Donde

$$\boldsymbol{\hat{\psi} = \sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(\hat{a}_{r(p)}u_{r(p)}e^{-ipx} + \hat{d}_r{}^\dagger{}_{(p)}v_{r(p)}e^{ipx}\right)}$$

$$\boldsymbol{\hat{\psi}^\dagger = \sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(\hat{a}_r{}^\dagger{}_{(p)}u_r{}^\dagger{}_{(p)}e^{ipx} + \hat{d}_{r(p)}v_r{}^\dagger{}_{(p)}e^{-ipx}\right)}$$

En el capítulo 8 del curso de Javier habíamos visto que, para los espinores del electrón:

$$\begin{pmatrix} p_0 - m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -p_0 - m \end{pmatrix} u_{r(p)} = 0$$

$$\begin{pmatrix} p_0 - m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -p_0 - m \end{pmatrix} = \begin{pmatrix} p_0 & 0 \\ 0 & -p_0 \end{pmatrix} - \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} + \begin{pmatrix} 0 & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{pmatrix} = p_0\gamma^0 - m + \begin{pmatrix} 0 & -\sigma^a p_a \\ \sigma^a p_a & 0 \end{pmatrix}$$

$$\begin{pmatrix} p_0 - m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -p_0 - m \end{pmatrix} = p_0\gamma^0 - m + \gamma^a p_a$$

$$(p_0\gamma^0 - m + \gamma^a p_a)u_{r(p)} = 0$$

$$(-m + \gamma^a p_a)u_{r(p)} + p_0\gamma^0 u_{r(p)} = 0$$

$$(-\gamma^a p_a + m)u_{r(p)} = p_0\gamma^0 u_{r(p)}$$

Multiplicando por γ^0 a ambos lados de la ecuación:

[1] $$(-\gamma^0\gamma^a p_a + \gamma^0 m)u_{r(p)} = p_0 u_{r(p)} = E_p u_{r(p)}$$

Para los espinores del positrón también es válida:

$$\begin{pmatrix} p_0 - m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -p_0 - m \end{pmatrix} v_{r(p)} = 0$$

Que resulta en:

$$(-\gamma^0\gamma^a p_a + \gamma^0 m)v_{r(p)} = p_0 v_{r(p)}$$

Pero en este caso, como la energía sería negativa, para lograr un valor "físico" (cap. 8, curso QED de Javier, 10:27) se cambia el signo de la energía:

[2] $$(-\gamma^0\gamma^a p_a + \gamma^0 m)v_{r(p)} = -E_p v_{r(p)}$$

También sabemos por el principio de correspondencia (cap. 1 del curso, minuto 9:06) que $p_a = i\partial_a$ que aplicando en [1] y [2] resulta:

[3] $(-i\,\gamma^0\gamma^a\partial_a + \gamma^0 m)u_{r\,(p)} = E_p u_{r\,(p)}$

[4] $(-i\,\gamma^0\gamma^a\partial_a + \gamma^0 m)v_{r\,(p)} = -E_p v_{r\,(p)}$

Con esto desarrollamos la segunda parte de la integral de $\hat{H} = \int d^3x\,\hat{\psi}^\dagger(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi}$

$$(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi} = (-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(\hat{a}_{r\,(p)}u_{r\,(p)}e^{-ipx} + \hat{d}_r{}^\dagger{}_{(p)}v_{r\,(p)}e^{ipx}\right)$$

Podemos conmutar los operadores con los espinores porque, como dice Javier en los comentarios de video del capítulo *"si los objetos vivieran en el mismo espacio, efectivamente habría que cambiar el orden. Sin embargo, los operadores de creación/destrucción actúan sobre el espacio de Hilbert de los estados de base |n> y los espinores "viven" en el espacio interno (de 4 dimensiones). Es decir, "viven" en espacios diferentes, por tanto conmutan"*.

$$(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi} = \sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\Big((-i\gamma^0\gamma^a\partial_a + \gamma^0 m)u_{r\,(p)}\hat{a}_{r\,(p)}e^{-ipx} + (-i\gamma^0\gamma^a\partial_a + \gamma^0 m)v_{r\,(p)}\hat{d}_r{}^\dagger{}_{(p)}e^{ipx}\Big)$$

Aplicando [3] y [4]:

$$(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi} = \sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(E_p u_{r\,(p)}\hat{a}_{r\,(p)}e^{-ipx} - E_p v_{r\,(p)}\hat{d}_r{}^\dagger{}_{(p)}e^{ipx}\right)$$

Y reemplazando en el Hamiltoniano:

$$\hat{H} = \int d^3x\,\hat{\psi}^\dagger(-i\gamma^0\gamma^a\partial_a + \gamma^0 m)\hat{\psi}$$

$$\hat{H} = \int d^3x \sum_{s=1}^{2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\left(\hat{a}_s{}^\dagger{}_{(k)}u_s{}^\dagger{}_{(k)}e^{ikx} + \hat{d}_{s\,(k)}v_s{}^\dagger{}_{(k)}e^{-ikx}\right)\sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\Big(u_{r\,(p)}\hat{a}_{r\,(p)}e^{-ipx} - v_{r\,(p)}\hat{d}_r{}^\dagger{}_{(p)}e^{ipx}\Big)$$

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2}\int d^3x\left(\hat{a}_s{}^\dagger{}_{(k)}u_s{}^\dagger{}_{(k)}e^{ikx} + \hat{d}_{s\,(k)}v_s{}^\dagger{}_{(k)}e^{-ikx}\right)\Big(u_{r\,(p)}\hat{a}_{r\,(p)}e^{-ipx} - v_{r\,(p)}\hat{d}_r{}^\dagger{}_{(p)}e^{ipx}\Big)$$

Trabajamos sobre la integral del espacio:

$$\int d^3x\left(\hat{a}_s{}^\dagger{}_{(k)}u_s{}^\dagger{}_{(k)}e^{ikx} + \hat{d}_{s\,(k)}v_s{}^\dagger{}_{(k)}e^{-ikx}\right)\left(u_{r\,(p)}\hat{a}_{r\,(p)}e^{-ipx} - v_{r\,(p)}\hat{d}_r{}^\dagger{}_{(p)}e^{ipx}\right) =$$

$$= \int d^3x \Big(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} e^{ikx}\, u_{r(p)} \hat{a}_{r(p)} e^{-ipx} + \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} e^{-ikx} u_{r(p)} \hat{a}_{r(p)} e^{-ipx}$$
$$- \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} e^{ikx} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{ipx} - \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} e^{-ikx} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{ipx} \Big) =$$

$$= \int d^3x \Big(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} e^{-i(p-k)x} + \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} e^{-i(p+k)x}$$
$$- \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{i(p+k)x} - \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{-i(k-p)x} \Big) =$$

$$= \int d^3x \left(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} e^{-i(p-k)x} \right) + \int d^3x \left(\hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} e^{-i(p+k)x} \right)$$
$$- \int d^3x \left(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{i(p+k)x} \right) - \int d^3x \left(\hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{-i(k-p)x} \right) =$$

Recordando que $i(p+k)x = i\left((E_p + E_k)t - (\vec{p} + \vec{k})\vec{x} \right)$

$$= \int d^3x \left(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} e^{-i(E_p - E_k)t + i(\vec{p}-\vec{k})\vec{x}} \right)$$
$$+ \int d^3x \left(\hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} e^{-i(E_p + E_k)t + i(\vec{p}+\vec{k})\vec{x}} \right)$$
$$- \int d^3x \left(\hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{i(E_p + E_k)t - i(\vec{p}+\vec{k})\vec{x}} \right)$$
$$- \int d^3x \left(\hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} e^{-i(E_k - E_p)t + i(\vec{k}-\vec{p})\vec{x}} \right) =$$

$$= e^{-i(E_p - E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} \int d^3x \left(e^{i(\vec{p}-\vec{k})\vec{x}} \right)$$
$$+ e^{-i(E_p + E_k)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} \int d^3x \left(e^{i(\vec{p}+\vec{k})\vec{x}} \right)$$
$$- e^{i(E_p + E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} \int d^3x \left(e^{-i(\vec{p}+\vec{k})\vec{x}} \right)$$
$$- e^{-i(E_k - E_p)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} \int d^3x \left(e^{i(\vec{k}-\vec{p})\vec{x}} \right) =$$

$$= e^{-i(E_p - E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} (2\pi)^3 \delta^3{}_{(p-k)} + e^{-i(E_p + E_k)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} (2\pi)^3 \delta^3{}_{(p+k)}$$
$$- e^{i(E_p + E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} (2\pi)^3 \delta^3{}_{(p+k)}$$
$$- e^{-i(E_k - E_p)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} (2\pi)^3 \delta^3{}_{(k-p)} =$$

Reemplazamos en el Hamiltoniano:

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{\sqrt{2E_p}} \sum_{s=1}^{2} \sum_{r=1}^{2} \Big(e^{-i(E_p - E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)}\, u_{r(p)} \hat{a}_{r(p)} (2\pi)^3 \delta^3{}_{(p-k)}$$
$$+ e^{-i(E_p + E_k)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} u_{r(p)} \hat{a}_{r(p)} (2\pi)^3 \delta^3{}_{(p+k)}$$
$$- e^{i(E_p + E_k)t} \hat{a}_s{}^\dagger{}_{(k)} u_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} (2\pi)^3 \delta^3{}_{(p+k)}$$
$$- e^{-i(E_k - E_p)t} \hat{d}_{s(k)} v_s{}^\dagger{}_{(k)} v_{r(p)} \hat{d}_r{}^\dagger{}_{(p)} (2\pi)^3 \delta^3{}_{(k-p)} \Big)$$

Analizamos cada término, cambiando de orden las integrales:

$$\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p-E_k)t}\hat{a}_s{}^{\dagger}{}_{(k)}u_s{}^{\dagger}{}_{(k)}\, u_{r(p)}\hat{a}_{r(p)}(2\pi)^3\delta^3{}_{(p-k)}$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p-E_p)t}\hat{a}_s{}^{\dagger}{}_{(p)}u_s{}^{\dagger}{}_{(p)}\, u_{r(p)}\hat{a}_{r(p)}(2\pi)^3$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} \hat{a}_s{}^{\dagger}{}_{(p)}u_s{}^{\dagger}{}_{(p)}\, u_{r(p)}\hat{a}_{r(p)}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p+E_k)t}\hat{d}_{s(k)}v_s{}^{\dagger}{}_{(k)}u_{r(p)}\hat{a}_{r(p)}(2\pi)^3\delta^3{}_{(p+k)}$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p+E_{-p})t}\hat{d}_{s(-p)}v_s{}^{\dagger}{}_{(-p)}u_{r(p)}\hat{a}_{r(p)}(2\pi)^3$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p+E_{-p})t}\hat{d}_{s(-p)}v_s{}^{\dagger}{}_{(-p)}u_{r(p)}\hat{a}_{r(p)}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{i(E_p+E_k)t}\hat{a}_s{}^{\dagger}{}_{(k)}u_s{}^{\dagger}{}_{(k)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}(2\pi)^3\delta^3{}_{(p+k)}$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{i(E_p+E_{-p})t}\hat{a}_s{}^{\dagger}{}_{(-p)}u_s{}^{\dagger}{}_{(-p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}(2\pi)^3$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{i(E_p+E_{-p})t}\hat{a}_s{}^{\dagger}{}_{(-p)}u_s{}^{\dagger}{}_{(-p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2E_k}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_k-E_p)t}\hat{d}_{s(k)}v_s{}^{\dagger}{}_{(k)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}(2\pi)^3\delta^3{}_{(k-p)}$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p-E_p)t}\hat{d}_{s(p)}v_s{}^{\dagger}{}_{(p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}(2\pi)^3$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} \hat{d}_{s(p)}v_s{}^{\dagger}{}_{(p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}$$

Volviendo al Hamiltoniano:

$$\begin{aligned}\hat{H} = &\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} \hat{a}_s{}^{\dagger}{}_{(p)}u_s{}^{\dagger}{}_{(p)}\, u_{r(p)}\hat{a}_{r(p)} \\ &+ \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{-i(E_p+E_{-p})t}\hat{d}_{s(-p)}v_s{}^{\dagger}{}_{(-p)}u_{r(p)}\hat{a}_{r(p)} \\ &- \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_{-p}}}\sum_{s=1}^{2}\sum_{r=1}^{2} e^{i(E_p+E_{-p})t}\hat{a}_s{}^{\dagger}{}_{(-p)}u_s{}^{\dagger}{}_{(-p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)} \\ &- \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_p}}\sum_{s=1}^{2}\sum_{r=1}^{2} \hat{d}_{s(p)}v_s{}^{\dagger}{}_{(p)}v_{r(p)}\hat{d}_r{}^{\dagger}{}_{(p)}\end{aligned}$$

Del formulario de Crul, cap. 43, se obtiene:
(https://crul.github.io/CursoTeoriaCuanticaDeCamposJavierGarcia/#capitulo-43)

${u_s}^{\dagger}{}_{(p)}\, {u_r}_{(p)} = 2E_p\delta_{rs}$ (ver fórmula 43.5 y explicación cap. 9 minuto 10:46)

${v_s}^{\dagger}{}_{(-p)} {u_r}_{(p)} = 0$ (ver fórmula 43.5 y ejercicio 43.6.c))

${u_s}^{\dagger}{}_{(-p)} {v_r}_{(p)} = 0$ (ver fórmula 43.5 y ejercicio 43.6.c))

${v_s}^{\dagger}{}_{(p)} {v_r}_{(p)} = 2E_p\delta_{rs}$ (ver fórmula 43.5 y explicación cap. 9 minuto 10:46)

$$\hat{H} = \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{\sqrt{2E_p}} \frac{1}{\sqrt{2E_p}} \sum_{s=1}^{2}\sum_{r=1}^{2} {\hat{a}_s}^{\dagger}{}_{(p)} 2E_p\delta_{rs} {\hat{a}_r}_{(p)} + 0 - 0$$
$$- \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{\sqrt{2E_p}} \frac{1}{\sqrt{2E_p}} \sum_{s=1}^{2}\sum_{r=1}^{2} {\hat{d}_s}_{(p)} 2E_p\delta_{rs}\, {\hat{d}_r}^{\dagger}{}_{(p)}$$

$$\hat{H} = \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{\sqrt{2E_p}} \frac{1}{\sqrt{2E_p}} 2E_p \sum_{s=1}^{2}\sum_{r=1}^{2} \delta_{rs} \left({\hat{a}_s}^{\dagger}{}_{(p)} {\hat{a}_r}_{(p)} - {\hat{d}_s}_{(p)} {\hat{d}_r}^{\dagger}{}_{(p)} \right)$$

$$\boxed{\boldsymbol{\hat{H} = \int \frac{d^3p}{(2\pi)^3} E_p \sum_{r=1}^{2} \left({\hat{a}_s}^{\dagger}{}_{(p)} {\hat{a}_r}_{(p)} - {\hat{d}_s}_{(p)} {\hat{d}_r}^{\dagger}{}_{(p)} \right)}}$$